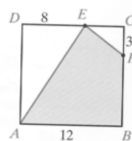
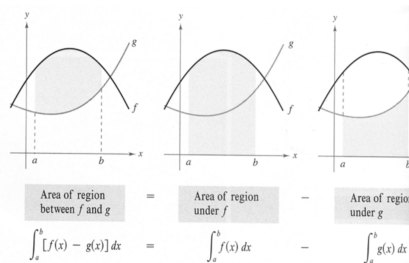


# DO NOW

How do you find the area of the shaded region at right?



## 7.1 Area of a Region Between Two Curves



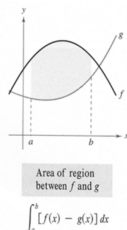
Page 1

Page 2

### Area of a Region Between Two Curves:

If  $f$  and  $g$  are continuous on  $[a, b]$  and  $g(x) \leq f(x)$  for all  $x$  in  $[a, b]$ , then the area of the region bounded by the graphs of  $f$  and  $g$  and the vertical lines  $x = a$  and  $x = b$  is:

$$A = \int_a^b [f(x) - g(x)] dx$$

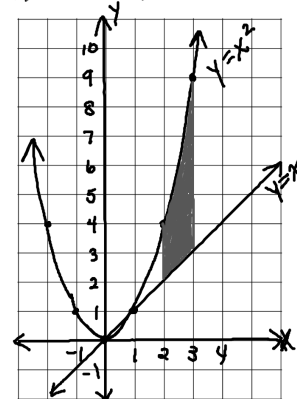


\*\*\* Make sure you know which function is above.

Example:

- Find the area bounded by  $y = x$ ,  $y = x^2$ ,  $x = 2$ , and  $x = 3$

$$\begin{aligned} A &= \int_2^3 [x^2 - x] dx \\ &= \left[ \frac{1}{3}x^3 - \frac{1}{2}x^2 \right]_2^3 \\ &= \left( \frac{1}{3} \cdot 3^3 - \frac{1}{2} \cdot 3^2 \right) - \left( \frac{1}{3} \cdot 2^3 - \frac{1}{2} \cdot 2^2 \right) \\ &= 9 - \frac{9}{2} - \frac{8}{3} + 2 \\ &= 11 - \frac{27}{6} - \frac{16}{6} \\ &= \frac{66}{6} - \frac{43}{6} \\ &= \frac{23}{6} = 3.8\bar{3} \end{aligned}$$



Page 3

Page 4

- Find the area between  $f(x) = 2 - x^2$  and  $g(x) = x$ .

\* You must find where they intersect to find bounds.

Intersections:

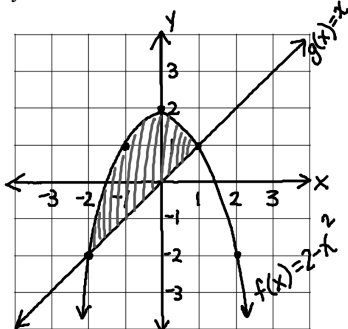
$$2 - x^2 = x$$

$$0 = x^2 + x - 2$$

$$0 = (x+2)(x-1)$$

$$x = -2, 1$$

$$\begin{aligned} &\int_{-2}^1 [2 - x^2 - x] dx \\ &= \left[ 2x - \frac{1}{3}x^3 - \frac{1}{2}x^2 \right]_{-2}^1 \\ &= \left( 2 - \frac{1}{3} - \frac{1}{2} \right) - \left( -4 + \frac{8}{3} - 2 \right) \\ &= 2 - \frac{5}{6} + 6 - \frac{8}{3} \\ &= \frac{48}{6} - \frac{5}{6} - \frac{16}{6} = \frac{27}{6} = 4.5 \end{aligned}$$



- Find the area bounded by  $f(x) = 3x^3 - x^2 - 10x$  and  $g(x) = -x^2 + 2x$

Intersections:

$$3x^3 - x^2 - 10x = -x^2 + 2x$$

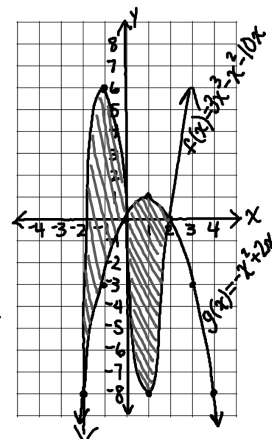
$$3x^3 - 12x = 0$$

$$3x(x^2 - 4) = 0$$

$$3x(x+2)(x-2) = 0$$

$$x = -2, 0, 2$$

$$\int_{-2}^0 [f(x) - g(x)] dx + \int_0^2 [g(x) - f(x)] dx$$



Page 5

Page 6

$$\begin{aligned}
 & \int_{-2}^0 [(3x^3 - x^2 - 10x) - (-x^2 + 2x)] dx + \int_0^2 [(-x^2 + 2x) - (3x^3 - x^2 - 10x)] dx \\
 & \int_{-2}^0 (3x^3 - 12x) dx + \int_0^2 (-3x^3 + 12x) dx \\
 & \left[ \frac{3}{4}x^4 - 6x^2 \right]_{-2}^0 + \left[ -\frac{3}{4}x^4 + 6x^2 \right]_0^2 \\
 & \left[ 0 - \left( \frac{3}{4}(16) + 24 \right) \right] + \left[ -\frac{3}{4}(16) + 24 \right] \\
 & -12 + 24 - 12 + 24 \\
 & \boxed{24}
 \end{aligned}$$

Page 7

# HOMEWORK

pg 452; 1 - 11 odd, 17 - 23 odd

Page 8